effect of changing the period, δ, from 0.25 to 0.5. A small decrement in heat transfer is observed in these cases. The maximum decrement obtained was about 4%.

In summary, the roughness elements are generally found to increase the heat transfer rate across the enclosure both at low and high Rayleigh numbers. The enhancement is more at low Rayleigh numbers in comparison with the cases with higher Rayleigh numbers. The maximum heat transfer enhancement (with respect to a corresponding smooth-walled enclosure) in this study is found to be 57%. This is obtained at $Ra_H = 2\times10^3$ for an enclosure with $A = 0.375$ and $\delta = 0.25$. In a transitional range between the conduction-dominated and convection-dominated cases, the presence of roughness elements delays the onset of convection motion. This effect reduces the heat transfer rate across the enclosure. For an enclosure with $A = 0.125$, the roughness elements generate a fluid motion with four cells at higher Rayleigh numbers. This effect drastically reduces the heat transfer rate across the enclosure. The maximum heat transfer decrement (with respect to a corresponding smooth-walled enclosure) in this geometry is found to be 61%, at a value of $Ra_H = 3\times10^4$.

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Correlations for simultaneously developing laminar flow and heat transfer in a circular tube

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INTRODUCTION

SIMULTANEOUSLY developing laminar flow and heat transfer in ducts has been widely analyzed in the past (see review by Shah and London [1]). More recent papers have shown that for accurate numerical solution of the classical Graetz problem (e.g. Conley *et al.* [2] and Poirier and Mujumdar [3]) or for simultaneously developing flow and heat transfer, Jensen [4], a very fine grid is needed to be concentrated at the tube inlet and wall where large gradients occur. Thus, most of the previous numerical studies are questionable at small non-dimensional distances because a variety of simplifications and coarse grids were used to obtain the solutions. Because the entrance length is significant for large Prandtl numbers ($Pr \geqslant 50$), accurate entrance length correlations for heat transfer and pressure drop are needed. However, very few correlations are available that can predict the local and/or average quantities. For example, the Churchill and Ozoe correlation [5], although covering the complete Pr and z^+ range, can predict only the local Nusselt number for the two limiting cases of constant wall temperature and constant wall heat flux, and its accuracy is known to degenerate for larger z^+ [1]. Therefore, the present investigation was initiated (i) to obtain accurate local and average Nusselt numbers for laminar flow through a straight circular tube with the general convective boundary condition, particularly very close to the entrance, and (ii) to develop accurate correlations for both local and average Nusselt numbers and friction factors, covering the complete Pr, Bi, and z^+ range.

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ANALYSIS

The non-dimensional governing equations for simultaneously developing laminar flow and heat transfer in a circular tube are:

$$\frac{1}{R}\frac{\partial}{\partial R}(RV_r)+\frac{\partial V_z}{\partial z^*}=0 \quad (1)$$

$$V_r\frac{\partial V_z}{\partial R}+V_z\frac{\partial V_z}{\partial z^*}=-\frac{\partial P}{\partial z^*}+\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial V_z}{\partial R}\right) \quad (2)$$

$$V_r\frac{\partial \Theta}{\partial R}+V_z\frac{\partial \Theta}{\partial z^*}=\frac{1}{Pr}\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial \Theta}{\partial R}\right). \quad (3)$$

NOMENCLATURE

- A tube cross-sectional area [m^2]
- Bi Biot number, $h_e D/k$
- D tube diameter [m]
- f Fanning friction factor
- h heat transfer coefficient [$W\ m^{-2}\ K^{-1}$]
- k thermal conductivity [$W\ m^{-1}\ K^{-1}$]
- L_h, L_{th} hydrodynamic entrance length; thermal entrance length [m]
- m grid expansion factor
- N number of radial nodes
- Nu Nusselt number, hD/k
- p, P pressure [Pa]; non-dimensional pressure, $p/\rho v_{in}^2$
- Pr Prandtl number
- r, R radial coordinate [m]; non-dimensional radial coordinate r/D
- Re Reynolds number, $v_{in}D/\nu$
- T temperature [K]
- v, v_r, v_z velocity, radial and axial velocity components, respectively [$m\ s^{-1}$]
- V_r, V_z v_z/v_{in}, $v_r D/\nu$
- z, z^*, z^+ axial coordinate [m], $z/(D\,Re)$, $z/(D\,Re\,Pr)$.

Greek symbols
- Θ $(T-T_{in})/(T_\infty - T_{in})$
- ν kinematic viscosity [$m^2\ s^{-1}$]
- ρ fluid density [$kg\ m^{-3}$]
- $\phi(i)$ fraction of tube radius to node center.

Subscripts
- b bulk
- e external
- in inlet
- m length-averaged
- T constant wall temperature boundary condition
- w wall
- z local quantity
- ∞ ambient.

These equations were solved subject to the assumptions of: (i) steady state, (ii) axisymmetric, (iii) constant fluid properties, (iv) negligible viscous dissipation, and (v) negligible axial conduction, and with the boundary conditions:

$$V_r(R,0) = 0,\quad V_z(R,0) = 1,\quad V_r(0.5,z^*) = V_z(0.5,z^*) = 0,$$

$$\frac{\partial V_z}{\partial R}(0,z^*) = 0,\quad \Theta(R,0) = 0,\quad \frac{\partial \Theta}{\partial R}(0,z^*) = 0,$$

and

$$\frac{\partial \Theta}{\partial R}(0.5,z^*) = Bi(1-\Theta_w). \qquad (4)$$

The last boundary condition involving Bi is the external convective boundary condition. With $Bi = 0$, the constant wall heat flux boundary condition is obtained; $Bi = \infty$ represents the constant wall temperature boundary condition. The above set of equations were solved using the general purpose commercial program 'PHOENICS', which is based on the SIMPLEST algorithm [6]. For the radial direction, a power law grid was generated by $\phi(i) = 1-[(N-i)/N]^m$, and the distance between two consecutive axial nodes was varied as $\Delta z^*_{j+1} = 1.1\Delta z^*_j$.

From the computed non-dimensional velocity and temperature profiles, the local Fanning friction factor and Nusselt number were calculated, respectively, using

$$f_z Re = 2\frac{\partial V_z}{\partial R}(0.5,z^*) \qquad (5)$$

$$Nu_z = \left(-\frac{\partial \Theta}{\partial R}(0.5,z^*)\right)\Big/(\Theta_w - \Theta_b) \qquad (6)$$

where

$$\Theta_b = \left(\int_A V_z \Theta\, dA\right)\Big/\left(\int_A V_z\, dA\right). \qquad (7)$$

The length-averaged friction factor and Nusselt number were obtained by integration of the respective local quantities over the tube length using Simpson's rule.

RESULTS AND DISCUSSION

To establish grid independence for z^+ as small as 10^{-6}, numerical experiments were carried out for $Pr = 0.7$, $Bi = \infty$, and $N = 60$, 120, 240, 480, 600, and 960 using $m = 1.5$. Increasing the number of radial nodes beyond 600 produced an insignificant change in the result. Hence, a grid expansion factor of 1.5 and 600×154 (radial $\times$ axial) nodes were used for all subsequent calculations. Results are reported (see Figs. 1–3) for $Pr = 0.7$, 5.0, 50.0, 500 and $Bi = 0$, 1, 3, 9, 20, and ∞ in the range $10^{-6} \leqslant z^+ \leqslant 0.2$. The initial value of z^+ was chosen as 10^{-8} in order to obtain accurate results in the region very close ($z^+ = 10^{-6}$) to the entrance especially for high Pr fluids; smaller values were avoided for computational economy. The present results for constant wall temperature and thermally developing flow were compared against the classical Graetz solution. In general, the present results were 0.1–0.4% higher than the analytic solution [1]. As a further check, the fully developed Nusselt numbers for a wide range of Bi were compared with the results of Sparrow and Patankar [7]. The agreement was within 0.2%.

The local and length-averaged friction factors were correlated, respectively, as:

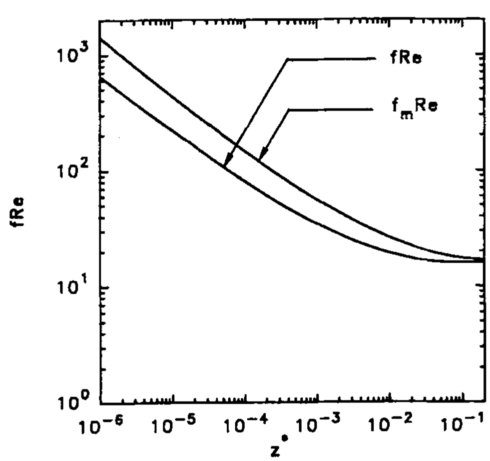
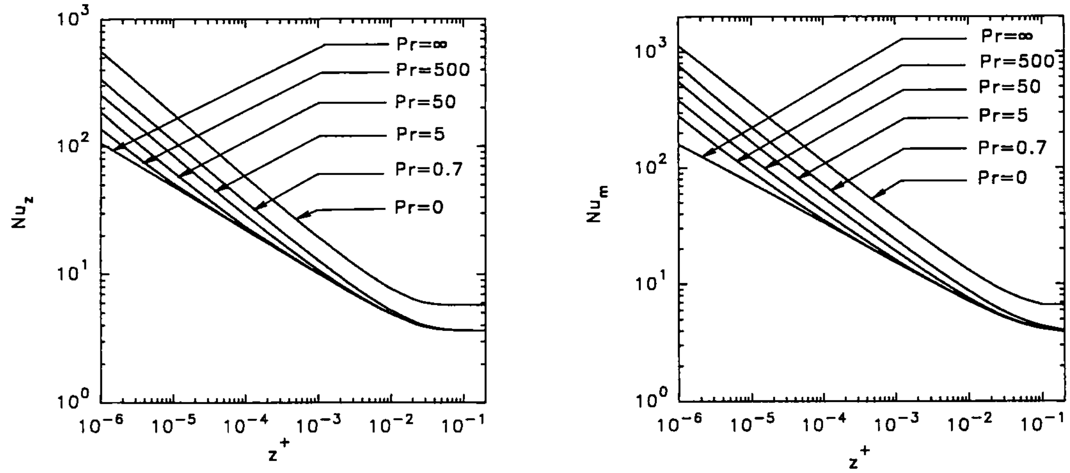


FIG. 1. Local and length-averaged Fanning friction factors for simultaneously developing laminar flow in a circular tube.



(a) (b)

FIG. 2. (a) Local Nusselt numbers for simultaneously developing laminar flow in a circular tube with constant wall temperature. (b) Length-averaged Nusselt numbers for simultaneously developing laminar flow in a circular tube with constant wall temperature.

$$\frac{f_z Re}{16} = \begin{cases} 1+\dfrac{3.44}{(z^*)^{0.5004}}-\dfrac{3.4109}{(z^*)^{0.5}}-\dfrac{0.00956}{(10^3 z^*)}, & 10^{-6} \leqslant z^* < 10^{-3} \\ 1+\dfrac{3.44}{(z^*)^{0.5035}}-\dfrac{3.4692}{(z^*)^{0.5}}-\dfrac{0.5881}{(10^3 z^*)}, & z^* \geqslant 10^{-3} \end{cases} \quad (8)$$

$$\frac{f_m Re}{16} = \begin{cases} 1+\dfrac{3.44}{(z^*)^{0.5001}}-\dfrac{3.3628}{(z^*)^{0.5}}+\dfrac{0.00432}{(10^3 z^*)}, & 10^{-6} \leqslant z^* < 10^{-3} \\ 1+\dfrac{3.44}{(z^*)^{0.5051}}-\dfrac{3.4464}{(z^*)^{0.5}}-\dfrac{1.1772}{(10^3 z^*)}, & z^* \geqslant 10^{-3} \end{cases} . \quad (9)$$

The above correlations predict the present results within $\pm 2.0\%$, except at $z^* = 10^{-6}$ where the local friction factor is under-predicted by 6.0%; the accuracy degenerates to $\pm(6 \sim 10)\%$ at small $z^* (< 10^{-5})$ if the digits beyond the second decimal place are truncated.

The hydrodynamic entrance length, L_h, is defined in this study as the length required for the local friction factor to fall to within 5% of its fully developed value. The present results gave a value of $L_h/(D\,Re) = 0.0287$.

The present results for the local Nusselt numbers for an arbitrary convective boundary condition were correlated as:

$$Nu_z = Nu_{z,T,Pr=\infty}\left(\frac{Nu_{z,Pr=\infty}}{Nu_{z,T,Pr=\infty}}\right)\left(\frac{Nu_z}{Nu_{z,Pr=\infty}}\right) \quad (10a)$$

where the correlation for the classical Graetz problem Nusselt number ($Nu_{z,T,Pr=\infty}$) is:

$$Nu_{z,T,Pr=\infty} = \begin{cases} -0.3856+1.022(z^+)^{-0.3366}, & 10^{-6} \leqslant z^+ \leqslant 10^{-3} \\ 3.6568+0.2249(z^+)^{-0.4956}\exp(-55.9857z^+), & z^+ > 10^{-3} \end{cases} \quad (10b)$$

the correlation for the Nusselt number ($Nu_{z,Pr=\infty}$) for thermally developing flow with an arbitrary value of Bi and $10^{-6} \leqslant z^+$ is:

$$\frac{Nu_{z,Pr=\infty}}{Nu_{z,T,Pr=\infty}} = \left[1+98.42\tanh\left(\frac{0.93}{Bi}\right)(1+z^{+\,-0.24})\right]^{0.03} \quad (10c)$$

and the Nusselt number (Nu_z) for an arbitrary Bi, $Pr \geqslant 0.7$, and $10^{-6} \leqslant z^+$ is:

$$\frac{Nu_z}{Nu_{z,Pr=\infty}} = \left[1+0.004\left(1+\tanh\frac{61.74}{Bi}\right)(z^+ Pr)^{-1.0}\right]^{0.12}. \quad (10d)$$

The above correlation (equation (10a)) predicts the present results for the complete range of Prandtl number ($Pr \geqslant 0.7$), Biot number ($\infty \geqslant Bi \geqslant 0$) and z^+ ($10^{-6} \leqslant z^+ \leqslant 0.2$) with a maximum error of 12% for $z^+ \leqslant 2.0 \times 10^{-6}$ and at the lowest Pr. The error drops sharply down to 3–4% in the mid range and is around 1% near the exit. Generally, the errors are lower for higher Prandtl numbers. The individual correlations (equations (10b–d)) each has a maximum error of less than 5% (very near the entrance) and usually less than 1%.

Similarly, the average Nusselt numbers were correlated as

$$Nu_m = Nu_{m,T,Pr=\infty}\left(\frac{Nu_{m,Pr=\infty}}{Nu_{m,T,Pr=\infty}}\right)\left(\frac{Nu_m}{Nu_{m,Pr=\infty}}\right) \quad (11a)$$

$$Nu_{m,T,Pr=\infty} = \begin{cases} -0.5632+1.571(z^+)^{-0.3351}, & 10^{-6} \leqslant z \leqslant 10^{-3} \\ 0.9828+1.129(z^+)^{-0.3686}, & 10^{-3} < z^+ \leqslant 10^{-2} \\ 3.6568+0.1272(z^+)^{-0.7373}\exp(-3.1563z^+), & z^+ > 10^{-2} \end{cases} \quad (11b)$$

$$\frac{Nu_{m,pr=\infty}}{Nu_{m,T,Pr=\infty}} = \left[1+209.92\tanh\left(\frac{1.12}{Bi}\right)(1+z^{+\,-0.20})\right]^{0.03}, \quad (11c)$$

and

$$\frac{Nu_m}{Nu_{m,Pr=\infty}} = \left[1+0.019\left(1+\tanh\frac{37.11}{Bi}\right)(z^+ Pr)^{-0.88}\right]^{0.17}. \quad (11d)$$

The general trend and the magnitude of the errors were similar to those for the local Nusselt numbers. Even though the Nusselt number for simultaneously developing flow and heat transfer (Figs. 2(a) and (b)) is a function of z^+ with Pr

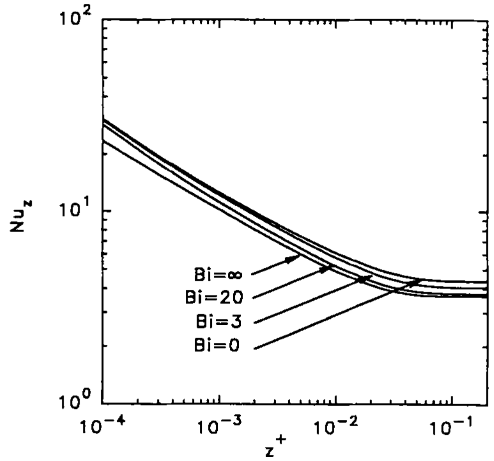


FIG. 3. Local Nusselt numbers for simultaneously developing laminar flow in a circular tube with convective boundary conditions and $Pr = 50.0$.

as a parameter, the Nusselt number ratios (equations (10d) and (11d)) are independent of Pr.

Thermal entrance length, L_{th}, is defined as the length required for the local Nusselt number to fall to within 5% of its fully developed value. The results indicated a very weak dependence of $L_{th}/(D\,Re\,Pr)$ on Pr, an observation consistent with that of Shah and London [1]. For $Pr > 1$, the thermal entrance length $(L_{th}/D\,Re\,Pr)$ was correlated within $\pm 2.5\%$ as:

$$L_{th}/D\,Re\,Pr = 0.0377\left(1+0.186\tanh\frac{4.018}{Bi}\right) \times (1+0.038Pr^{-4.98}). \quad (12)$$

CONCLUSIONS

Numerical simulation of simultaneously developing flow and heat transfer in a circular tube with convective boundary conditions has been carried out. Accurate local and average Nusselt numbers and friction factors were obtained and correlated in the region very close to the entrance. The analysis neglects effects which could be significant in real flows, such as tube wall axial conduction and property variation. Thus, the correlations presented are applicable to situations where these effects are negligible such as flow in thin walled high conductivity tubes with low heating rate/temperature difference. The results presented would be useful for assessing the influence of these effects in the entrance region very close to the inlet.

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